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Possible non-decoupling effects of heavy Higgs bosons in $e^+e^- \to W^+W^-$ within THDM

M. Malinský[1,2,a], J. Hořejší[1,b]

[1] IPNP, Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic
[2] S.I.S.S.A., Trieste, Italy

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Abstract. We discuss the origin of the non-decoupling effects of the heavy Higgs bosons within the two Higgs doublet extension (THDM) of the standard model (SM) and illustrate it by means of the one-loop calculation of the differential cross sections of the process $e^+e^- \to W^+W^-$ in both the decoupling and the non-decoupling regimes. We argue that there are many regions in the THDM parametric space in which the THDM and SM predictions differ by several percents and such effects could, at least in principle, be testable at the future experimental facilities.

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1 Introduction

Though having been on the market for more than thirty years, the two Higgs doublet model (THDM) [1] still provides one of the most viable extensions of the standard model (SM) and, surprisingly enough, the activity in this field seems to grow in the last decade – see e.g. [2–6] and references therein. It earns its popularity namely because of its capability to incorporate many sources of physics beyond SM [7, 8], be it CP-violation in the Higgs sector, additional contributions to the anomalous magnetic moment of the muon or simply the fact that the two Higgs doublet structure with five massive physical states (and at least one around the weak scale) mimics nicely many features of the minimal supersymmetric SM (MSSM). On top of that, as we shall see, the Higgs sector of THDM can exhibit some particular features which are completely absent in MSSM, namely the relatively large non-decoupling effects of the heavy Higgs bosons [9–16] which can arise once the heavy Higgs spectrum is sufficiently non-degenerate.

This paper is organized as follows: in Sect. 2 we present a general systematic discussion of the origin of various non-decoupling effects and the connection of their magnitude with the parameters of the Higgs potential and the shape of the heavy Higgs spectrum.

In Sects. 3 and 4 we use these results to estimate quantitatively the scale of the effects in question in the physical process of particular interest, namely $e^+e^- \to W^+W^-$. We thus extend the earlier analysis of Kanemura et al. [10] based on the equivalence theorem (ET) for longitudinal vector bosons [17], valid in the large s limit. Having in hand the results of our previous works [18, 19] in which we discussed the non-decoupling structures arising in the one-loop triple gauge vertices of THDM we can go far beyond the ET approximation.

Section 5 is devoted to several quantitative illustrations of the typical behavior of the relevant cross sections both in the decoupling and the non-decoupling regimes, in full agreement with the estimates.

2 THDM overview

Adopting the notation of [21] the most general form of the THDM Higgs potential reads

$$\begin{aligned}
V(\Phi_1,\Phi_2) &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 \qquad (1)\\
&- \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2\\
&+ \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right)\\
&+ \left\{\frac{\lambda_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right)\right.\\
&\qquad \left. + \text{h.c.}\right\}.
\end{aligned}$$

Here the Φ_1 and Φ_2 are two $SU(2)$ doublets with the same SM-like hypercharges. The neutral components of both of them can develop non-zero VEVs around the weak scale and break the electroweak symmetry downto the electromagnetic $U(1)$ in the usual manner.

[a] e-mail: malinsky@sissa.it
[b] e-mail: horejsi@ipnp.troja.mff.cuni.cz

The massive Higgses h^0, H^0, A^0 and $H^\pm$ as well as the Goldstone bosons G^0 and $G^\pm$ are then "encoded" within these doublets as

$$\Phi_1 = \frac{1}{\sqrt{2}} \times \begin{bmatrix} \sqrt{2}G^+\cos\beta - \sqrt{2}H^+\sin\beta \\ H^0\cos\alpha - h^0\sin\alpha + v_1 + \mathrm{i}G^0\cos\beta - \mathrm{i}A^0\sin\beta \end{bmatrix},$$

$$\Phi_2 = \frac{1}{\sqrt{2}} \times \begin{bmatrix} \sqrt{2}G^+\sin\beta + \sqrt{2}H^+\cos\beta \\ H^0\sin\alpha + h^0\cos\alpha + v_2 + \mathrm{i}G^0\sin\beta + \mathrm{i}A^0\cos\beta \end{bmatrix},$$

where as usual $\tan\beta = v_2/v_1$. For the sake of simplicity we take both v_1 and v_2 real. Next, the neutral scalar mixing angle α is given by

$$\cos^2(\alpha-\beta) = \frac{m_{\mathrm{L}}^2 - m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2}, \qquad (2)$$

where we have denoted

$$m_{\mathrm{L}}^2 \equiv \left[\lambda_1\cos^4\beta + \lambda_2\sin^4\beta + \frac{1}{2}\left(\lambda + 2D\sin^2\beta\right)\right]v^2,$$

with $\lambda \equiv \lambda_3 + \lambda_4 + \lambda_5$ and $D \equiv \lambda_7^{\mathrm{R}}\tan\beta + \lambda_6^{\mathrm{R}}\cot\beta$ (the superscript R denotes the real part).

2.1 The physical Higgs spectrum

Let us inspect the general formulae for the physical Higgs masses descending from the potential (1):

$$m_{h^0}^2 = \frac{1}{2}(1-\kappa)M^2 + \frac{v^2}{\cos 2\alpha} \times \left[B_2\sin^2\beta - A_1\cos^2\beta + \frac{1}{4}C(1+\cos 2\alpha\cos 2\beta)\right],$$

$$m_{H^0}^2 = \frac{1}{2}(1+\kappa)M^2 + \frac{v^2}{\cos 2\alpha} \times \left[A_2\cos^2\beta - B_1\sin^2\beta - \frac{1}{4}C(1-\cos 2\alpha\cos 2\beta)\right],$$

$$m_{A^0}^2 = M^2 - \frac{1}{2}\left(2\lambda_5^{\mathrm{R}} + \lambda_6^{\mathrm{R}}\cot\beta + \lambda_7^{\mathrm{R}}\tan\beta\right)v^2, \qquad (3)$$

$$m_{H^\pm}^2 = M^2 - \frac{1}{2}\left(\lambda_4 + \lambda_5^{\mathrm{R}} + \lambda_6^{\mathrm{R}}\cot\beta + \lambda_7^{\mathrm{R}}\tan\beta\right)v^2,$$

where

$$M^2 \equiv \frac{m_{12}^{2\ \mathrm{R}}}{\sin\beta\cos\beta}, \qquad \kappa \equiv -\frac{\cos 2\beta}{\cos 2\alpha},$$

$$A_1 \equiv \lambda_1\sin^2\alpha - \lambda_7^{\mathrm{R}}\tan\beta\cos^2\alpha,$$
$$A_2 \equiv \lambda_1\cos^2\alpha - \lambda_7^{\mathrm{R}}\tan\beta\sin^2\alpha,$$
$$B_1 \equiv \lambda_2\sin^2\alpha - \lambda_6^{\mathrm{R}}\cot\beta\cos^2\alpha,$$
$$B_2 \equiv \lambda_2\cos^2\alpha - \lambda_6^{\mathrm{R}}\cot\beta\sin^2\alpha,$$
$$C \equiv \lambda_7^{\mathrm{R}}\tan\beta - \lambda_6^{\mathrm{R}}\cot\beta.$$

Notice that the two mass parameters m_{11}^2 and m_{22}^2 were as usual fixed by the necessary conditions for the VEVs of Φ_1 and Φ_2 to minimize the potential. For $\lambda_6 = \lambda_7 = m_{12} = 0$ one recovers the formulae given previously in the literature, see e.g. [21, 22] and references therein.

Let us call *heavy Higgs mass limit* the situation, in which the masses of all the THDM Higgs bosons but h^0 are much larger than the weak scale.

One can see that there are in general two basic quantities responsible for the shape of the Higgs spectrum (3): the gauge singlet mass parameter M (alias m_{12}) and the VEV magnitude v. Since M is not protected by the gauge symmetry it could be naturally much larger than v and in such case the heavy Higgs mass limit is achieved entirely by enlarging M. On the other hand, there are unitarity bounds on the masses of the "heavy" members of the Higgs spectrum preventing them to be extremely heavy [23]. Moreover, the v is often accompained by (in principle) numerically large factors $\propto \lambda_7\tan\beta$ (or $\lambda_6\cot\beta$) which enhances some of the "λv^2 terms" obviating to large extent the necessity of having a dominant M to achieve the heavy Higgs mass limit.

This is in sharp contrast with the situation in the MSSM where only one free parameter μ is left to play with, because the quartic couplings in the Higgs potential are fixed by supersymmetry.

2.2 Non-decoupling regime and spectrum distorsions

Therefore, it is convenient to distinguish between two different modes in which the "heavy part" of the Higgs spectrum acquires the masses.

(1) If it is due to the dominance of the singlet mass terms (M-components) in the relations (3) let us call it *the decoupling regime*. Perhaps it is worth noting that although the only explicit mass present in (3) is m_{12} one should not forget about m_{11} and m_{22} that are "hidden" in particular combinations of the other parameters in the game which can to some extent mimic their role unless $\lambda_{6,7} = 0$ (see also the comments in Sect. 5.2) .

(2) The contributions coming from the M components are comparable with the other "λv^2" parts in (3); such situation is called *the non-decoupling regime.*

As the terminology suggests, in the decoupling regime the heavy Higgs bosons exhibit a decoupling behavior in accordance with the famous Appelquist–Carazzone theorem [24]. In this case one can easily show that the requirement of coincidence of the THDM h^0 with the SM Higgs boson η (with masses not far from m_W) and the relation (2) lead to $\kappa \sim 1$ and therefore *the heavy Higgs spectrum is quasidegenerate,* $m_{H^0} \sim m_{A^0} \sim m_{H^\pm} \sim M$.

On the other hand, in the non-decoupling regime *the heavy Higgs spectrum should be distorted* and one can in principle expect substantial effects in measurable quantities which should grow with the weights of the λv^2 terms in (3), i.e. with the magnitude of such a distortion. This was used as a non-trivial consistency check of the numerical results we present below.

From this point of view the behavior of the heavy Higgs bosons in the MSSM is very simple in comparison with THDM; in fact, in MSSM there is no such non-decoupling regime and all the heavy Higgs bosons therein should therefore tend to decouple from the weak scale physics. This was confirmed explicitly in [25].

This can be used e.g. as a simple heuristic explanation of what happens in [21, 22], where the considered non-decoupling effects of the heavy part of the Higgs spectrum tend to minimize provided a partial degeneration in the heavy Higgs sector is achieved ($m_{H^\pm} \to m_{A^0}$).

In the remaining part of this paper we demonstrate these principles in the particular case of physical cross sections of the process $e^-e^+ \to W^-W^+$ computed within the THDM framework at one-loop order in comparison with the SM predictions. We generalize the earlier work [10] beyond the ET approximation used therein. Among other things, our approach allows one to consider general configurations of polarizations of the final state vector bosons.

3 $\mathrm{d}\sigma(e^-e^+ \to W^-W^+)$ in THDM versus SM

Since the one-loop form of the differential cross section within the SM is very well known [26], we can use the similarity of THDM to simplify our life by dealing with the pieces of information which are specific for THDM, namely the contributions to the one-loop amplitude and the bremsstrahlung terms that are different for THDM and SM.

Therefore, it is reasonable to work with a quantity that measures just the deviation of the THDM and SM cross sections under consideration [10]; let us define it as

$$\delta \equiv \frac{\mathrm{d}\sigma^{\mathrm{THDM}}(e^+e^- \to W^+W^-)}{\mathrm{d}\sigma^{\mathrm{SM}}(e^+e^- \to W^+W^-)} - 1 \tag{4}$$

the two differential cross section in the previous expression can be written as

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{A}} + \mathrm{d}\sigma_{\mathrm{B}},$$

where the pieces $\mathrm{d}\sigma_{\mathrm{A}}$ come from the "amplitude-squared" terms

$$\mathrm{d}\sigma_{\mathrm{A}} = k_1 \left|\mathcal{M}\right|^2 \mathrm{dLips}, \tag{5}$$

while the $\mathrm{d}\sigma_{\mathrm{B}}$ terms represent the bremsstrahlung effects

$$\mathrm{d}\sigma_{\mathrm{B}} = k_2 \int \mathrm{d}k_\gamma \left|\mathcal{B}(k_\gamma, \ldots)\right|^2 \mathrm{dLips}. \tag{6}$$

Expanding now the THDM amplitude around the corresponding SM form

$$\mathcal{M}^{\mathrm{THDM}}_{\mathrm{tree}} = \mathcal{M}^{\mathrm{SM}}_{\mathrm{tree}} + \Delta\mathcal{M}_{\mathrm{tree}},$$

$$\mathcal{M}^{\mathrm{THDM}}_{\mathrm{loop}} = \mathcal{M}^{\mathrm{SM}}_{\mathrm{loop}} + \Delta\mathcal{M}_{\mathrm{loop}}$$

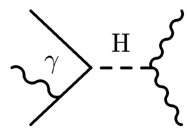
($\Delta\mathcal{M}_{\mathrm{tree}}$ and $\Delta\mathcal{M}_{\mathrm{loop}}$ are just the differences of the tree- and one-loop amplitudes respectively), one can recast the δ as

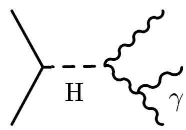
$$\delta = 2\mathrm{Re}\left[\frac{\Delta\mathcal{M}_{\mathrm{tree}} + \Delta\mathcal{M}_{\mathrm{loop}}}{\mathcal{M}^{\mathrm{SM}}_{\mathrm{tree}}}\right] + \frac{k_2}{k_1}\int \mathrm{d}k_\gamma \frac{\left|\mathcal{B}^{\mathrm{THDM}}\right|^2 - \left|\mathcal{B}^{\mathrm{SM}}\right|^2}{\left|\mathcal{M}^{\mathrm{SM}}_{\mathrm{tree}}\right|^2} + \ldots \tag{7}$$

Here the k_i are $\mathcal{O}(1)$ geometrical factors and the integration over k_γ covers the IR-singular piece of the phase space spanned by the photon momenta.

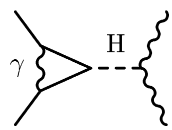
3.1 Bremsstrahlung terms

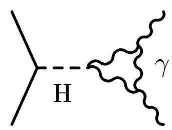
Let us first explore the bremsstrahlung terms in (7). Using the identity $|A|^2 - |B|^2 = \frac{1}{2}(A-B)(A+B)^* + \mathrm{h.c.}$ we see that the only terms that survive (i.e. those that are not common to both THDM and SM) come from the graphs (H denotes the generic neutral Higgses in the game):



and


These diagrams are necessary to regulate the singular infrared behavior of $\mathrm{d}\sigma_{\mathrm{A}}$ caused by the presence of



and


All the above graphs are IR-singular but the divergences cancel in the physical cross section formulae and once we specify the IR regulator scale they provide just logarithmic corrections to $\mathrm{d}\sigma_{\mathrm{A}}$. As we shall see, the presence of the electron Yukawa coupling makes these terms suppressed by factors of the order of $m_e/m_W \sim 10^{-5}$ with respect to the leading contributions to δ coming from the IR-regular terms in $\mathrm{d}\sigma_{\mathrm{A}}$. Thus, in the leading approximation we can omit the bremsstrahlung effects entirely.

3.2 The leading term

This means that we can forget about the second term of the expansion (7) and also keep only the IR-regular subset of graphs corresponding to $\mathcal{M}^{\mathrm{THDM}}_{\mathrm{loop}}$, which we shall denote by $\overline{\mathcal{M}}^{\mathrm{THDM}}_{\mathrm{loop}}$. Next, it is worth noting that the $\Delta\mathcal{M}_{\mathrm{tree}}$ in (7) also suffers of the "omnipresent" electron Yukawa coupling which again makes it much smaller in comparison with the leading loop contributions to $\mathcal{M}^{\mathrm{THDM}}_{\mathrm{loop}}$ coming (as we shall see) from the renormalization of the triple gauge vertices (TGV). To conclude, the leading contribution to δ can be written in the form

$$\delta = 2\mathrm{Re}\frac{\Delta\overline{\mathcal{M}}_{\mathrm{loop}}}{\mathcal{M}^{\mathrm{SM}}_{\mathrm{tree}}} + \ldots \tag{8}$$

4 Computation of $\Delta\overline{\mathcal{M}}_{\mathrm{loop}}$

Thus, all we need is only the sum of all the IR-safe one-loop diagrams which are not shared by the THDM and SM, i.e. those loop graphs which contain at least one Higgs propagator.

4.1 Renormalization scheme and gauge choice

We have decided to use the on-shell renormalization scheme [27] in which the renormalized masses parametrizing the one-loop quantities coincide with the physical masses of the fields in the game; this simplifies greatly the interpretation of various limits under consideration (in particular, we use the set of counterterms defined in [28]). At first glance, it might seem that this also obviates the necessity to deal with the renormalization of the external legs of Feynman graphs. In fact, this is not true because in the on-shell scheme non-trivial finite parts of various counterterms reappear and these bring back the self-energy diagrams. We will work in the Feynman ($\xi = 1$) gauge to have the IVB propagators as simple as possible. This of course requires one to take into account also the unphysical Goldstone bosons.

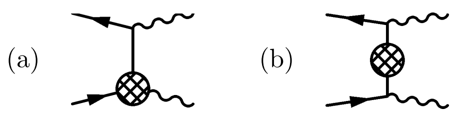
Note that there is no need to take care of the ghost fields (which do not decouple from the Higgs sector unless $\xi = 0$) because all the relevant topologies contributing to $\Delta\overline{\mathcal{M}}_{\mathrm{loop}}$ containing the ghost loop involve also the Yukawa couplings (one-loop irreducible graphs) or do not contribute substantially (oblique corrections).

4.2 Feynman diagrams contributing to $\Delta\overline{\mathcal{M}}_{\mathrm{loop}}$

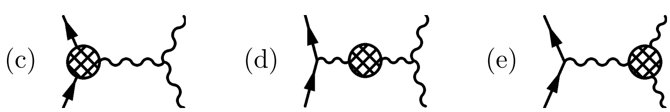
Let us first classify the topologies of Feynman diagrams contributing to $\Delta\overline{\mathcal{M}}_{\mathrm{loop}}$ that do not subtract trivially in (4), i.e. those that are not common to the two models. In what follows, the symbol  denotes a loop involving at least one Higgs propagator while  is used for loops built entirely from the other SM fields. In both cases also the corresponding counterterms must be taken into account.

Concerning the tree-level origin of the relevant one-loop graphs one can identify several subgroups of them, namely:

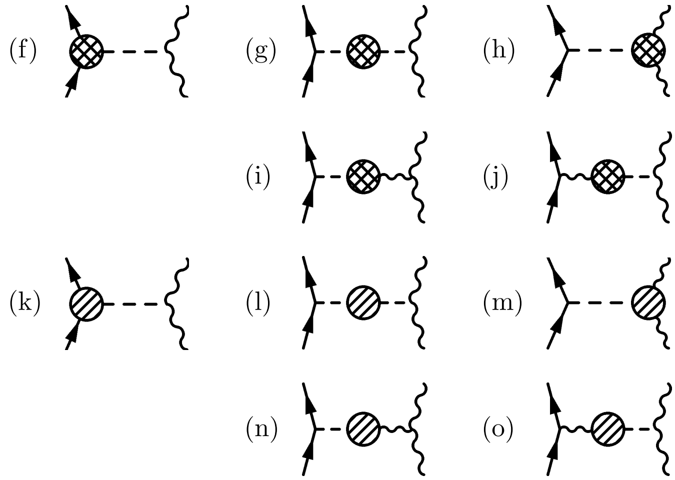
Neutrino in the t-channel:



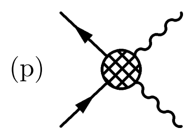
Z and γ in the s-channel:



Higgs in the s-channel:



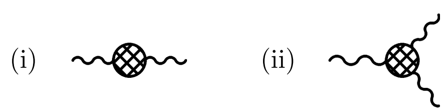
Box diagrams:



The total number of the graphs with the topologies displayed here is enormous. However, not all of them are relevant in the leading approximation.

4.3 Relevant topologies

As before, the presence of the electron Yukawa couplings allows one to neglect all graphs of the types (a), (b), (c), (f), (g), (h), (i), (l), (m) and (n) in comparison with the types (d), (e), (k), (o) and (p) where there is no such factor. Next, the blob in the type (k) provides just a correction to the Yukawa vertex which is fixed by the renormalization conditions to be comparable with the Yukawa coupling itself. The diagrams of type (o) are again m_e/m_W suppressed; the electron mass here arises from the Dirac equation. We are therefore left with the IVB "vacuum polarization" graphs in (d) (the oblique corrections to the IVB propagators), the corrections to the triple gauge vertices (e) and the UV convergent box diagrams (p). However, among them only those involving the Higgs bosons coupled to the IVB lines need to be taken into account, otherwise the small Yukawa coupling reappears. Next, since the boxes are UV-finite they must decouple trivially in the heavy Higgs mass limit described above. All that remain at the leading order are therefore the graphs of the type



that we shall treat separately.

4.4 Decoupling behavior of the oblique corrections

The renormalized inverse propagator of a massive vector boson is given by

$$\begin{aligned}\mathrm{i}\Gamma^{(1)}_{\mu\nu}(k) &= \mathrm{i}\Gamma^{(0)}_{\mu\nu}(k) + \mathrm{i}\Pi_{\mu\nu}(k) + \mathrm{i}(Z-1)k^2 P^{\mathrm{T}}_{\mu\nu} \\ &\quad -\mathrm{i}\delta m^2 g_{\mu\nu} + \mathrm{i}k^2 P^{\mathrm{L}}_{\mu\nu}\delta\alpha^{-1}, \qquad (9)\end{aligned}$$

where $P^{\mathrm{T}}_{\mu\nu} \equiv g_{\mu\nu} - k_\mu k_\nu k^{-2}$ and $P^{\mathrm{L}}_{\mu\nu} \equiv k_\mu k_\nu k^{-2}$ are the transverse and longitudinal projectors,

$$\mathrm{i}\Pi_{\mu\nu}(k) \equiv \mathrm{i}k^2 \Pi^{\mathrm{T}}(k^2) P^{\mathrm{T}}_{\mu\nu} + \mathrm{i}k^2 \Pi^{\mathrm{L}}(k^2) P^{\mathrm{L}}_{\mu\nu}$$

is the sum of all relevant one-loop graphs, $(Z-1)$ and δm^2 are the wave-function and mass counterterms and $\Gamma^{(0)}_{\mu\nu} = (k^2 - m^2) P^{\mathrm{T}}_{\mu\nu} + \alpha^{-1}\left(k^2 - \alpha m^2\right) P^{\mathrm{L}}_{\mu\nu}$ is the tree-level massive gauge boson propagator. In the on-shell scheme the counterterms are fixed by [27]

$$\Gamma^{\mathrm{T}}(k^2 = m^2) = 0, \qquad \left.\frac{\mathrm{d}}{\mathrm{d}k^2}\right|_{k^2=m^2} \Gamma^{\mathrm{T}}(k^2) = 1, \qquad (10)$$

which yields

$$\begin{aligned} Z - 1 &= -\left[\Pi^{\mathrm{T}}(m^2) + m^2 \Pi^{\mathrm{T}\prime}(m^2)\right], \\ \delta m^2 &= -m^4 \Pi^{\mathrm{T}\prime}(m^2). \end{aligned}$$

We do not need to deal with the longitudinal part of the renormalized IVB propagators because they typically produce the suppressing m_e/m_W factors. We can also omit all tadpole diagrams: their contributions to Π^{T} are proportional to k^{-2} and therefore cancel (as they should) in $Z-1$ and $[k^2\Pi^{\mathrm{T}}(k^2) - \delta m^2]$ in Γ^{T}.

The remaining graphs are those appearing in the massive scalar electrodynamics. It is already easy to show that such contributions to δ fall rapidly in the heavy Higgs mass limit regardless of the way the limit is achieved.

4.5 Triple gauge vertex corrections

Concerning the differences of the heavy Higgs corrections to the triple gauge vertices in THDM and in SM the major part of the work has already been done before [18], so let us mention just the important points. The differences of the relevant vertex functions can be written in the form

$$\begin{aligned}&\Delta\Gamma^{VWW}_{\sigma\mu\nu} \\ &= \sum_{i=1}^{3}\left(\Delta\delta Z_{TGV} + \Delta\Pi^{VWW}_i\right) C^i_{\sigma\mu\nu} + \sum_{i=4}^{7} \Delta\Pi^{VWW}_i C^i_{\sigma\mu\nu} \\ &\qquad + \mathrm{sym.},\end{aligned}$$

where

$$\begin{aligned}&C^1_{\sigma\mu\nu} \equiv q_{1\sigma} g_{\mu\nu}, \quad C^2_{\sigma\mu\nu} \equiv 2q_{2\mu} g_{\sigma\nu}, \quad C^3_{\sigma\mu\nu} \equiv q_{1\mu} g_{\sigma\nu}, \\ &C^4_{\sigma\mu\nu} \equiv \frac{1}{m_W^2} q_{1\sigma} q_{1\mu} q_{1\nu}, \quad C^5_{\sigma\mu\nu} \equiv \frac{1}{m_W^2} q_{1\sigma} q_{1\mu} q_{2\nu}, \quad (11) \\ &C^6_{\sigma\mu\nu} \equiv \frac{1}{m_W^2} q_{1\sigma} q_{2\mu} q_{1\nu}, \quad C^7_{\sigma\mu\nu} \equiv \frac{1}{m_W^2} q_{2\sigma} q_{1\mu} q_{1\nu}\end{aligned}$$

are basic kinematical structures composed of the outgoing momenta $q_{1,2}$ of the $W^\pm$ bosons in the final state. Next,

$$\Delta\delta Z_{TGV} \equiv (\delta Z_{TGV})_{\mathrm{THDM}} - (\delta Z_{TGV})_{\mathrm{SM}} \qquad (12)$$

are the differences of the corresponding finite parts of counterterms Z_{TGV} (computed by means of the W self-energy diagrams) and

$$\Delta\Pi^{VWW}_i \equiv \left(\Pi^{VWW}_i\right)_{\mathrm{THDM}} - \left(\Pi^{VWW}_i\right)_{\mathrm{SM}} \qquad (13)$$

are the differences of the formfactors Π^{VWW}_i (descending from the triangle diagrams contributing at the one-loop level to the triple gauge vertices); for more details see [18].

4.6 δ revised

Armed by the information given in the last three subsections we are ready to write down the explicit formulae for the leading part of (8):

$$\begin{aligned}\Delta\overline{\mathcal{M}}_{\mathrm{loop}} &\doteq \sum_{V=\gamma,Z} g_{eeV} g_{VWW} \bar{v}(p_1)\gamma_\lambda u(p_2) \frac{-\mathrm{i}g^{\lambda\sigma}}{s - m_V^2} \\ &\quad \times \Delta\Gamma^{VWW}_{\sigma\mu\nu}(q_1,q_2)\,\varepsilon^{*\mu}(q_1)\varepsilon^{*\nu}(q_2). \qquad (14)\end{aligned}$$

Inspecting (11) and using the identities

$$q_1.\varepsilon(q_1) = q_2.\varepsilon(q_2) = 0\,,$$

we can write (denoting by A the set $(q_1^2, q_2^2, q_1.q_2)$ and correspondingly $B \equiv (q_2^2, q_1^2, q_2.q_1)$, which is nothing but the "sym." operation applied on A [18])

$$\begin{aligned}&\Delta\Gamma^{VWW}_{\sigma\mu\nu}(q_1,q_2)\,\varepsilon^{*\mu}(q_1)\varepsilon^{*\nu}(q_2) = \varepsilon^{*\mu}(q_1)\varepsilon^{*\nu}(q_2) \\ &\times\Big[\left(\Delta\Pi^{VWW}_1 + \Delta\delta Z_{TGV}\right)(A) q_{1\sigma} g_{\mu\nu} \\ &- \left(\Delta\Pi^{VWW}_1 + \Delta\delta Z_{TGV}\right)(B) q_{2\sigma} g_{\mu\nu} \\ &+ \left(\Delta\Pi^{VWW}_2 + \Delta\delta Z_{TGV}\right)(A) 2q_{2\mu} g_{\sigma\nu} \\ &- \left(\Delta\Pi^{VWW}_2 + \Delta\delta Z_{TGV}\right)(B) 2q_{1\nu} g_{\sigma\mu} \\ &+ \Delta\Pi^{VWW}_6(A)\frac{q_{1\sigma}q_{2\mu}q_{1\nu}}{m_W^2} - \Delta\Pi^{VWW}_6(B)\frac{q_{2\sigma}q_{1\nu}q_{2\mu}}{m_W^2}\Big].\end{aligned}$$

If, for simplicity, we take the final state W bosons on the mass shell, i.e. $q_1^2 = q_2^2 = m_W^2$ we get $A = B$ and (14) can be recast in the form

$$\begin{aligned}&\Delta\overline{\mathcal{M}}_{\mathrm{loop}} \doteq \sum_{V=\gamma,Z} g_{eeV} g_{VWW} \bar{v}(p_1)\gamma_\lambda u(p_2) \frac{-\mathrm{i}g^{\lambda\sigma}}{s - m_V^2} \\ &\times\Big[\left(\Delta\Pi^{VWW}_1 + \Delta\delta Z_{TGV}\right)\left(m_W^2, s\right)(q_1 - q_2)_\sigma g_{\mu\nu} \\ &+2\left(\Delta\Pi^{VWW}_2 + \Delta\delta Z_{TGV}\right)\left(m_W^2, s\right)(q_{2\mu} g_{\sigma\nu} - q_{1\nu} g_{\sigma\mu})\end{aligned}$$

$$+\Delta\Pi_6^{VWW}\left(m_W^2, s\right)\frac{q_{2\mu}q_{1\nu}}{m_W^2}(q_1-q_2)_\sigma\Big]\varepsilon^{*\mu}(q_1)\varepsilon^{*\nu}(q_2).$$

Next, the momentum conservation $q_1+q_2+p_1+p_2=0$ and the Dirac equations $\not{p}_i = m_e$ allow for further simplifications. Plugging in the coupling constants given in [18] one finally arrives at

$$\begin{aligned}\Delta\overline{\mathcal{M}}^{\mathrm{LR}}_{\mathrm{loop}} \doteq -\mathrm{i}e^2 2\Bigg\{\frac{1}{s}\Bigg[&\mathcal{M}_2^+\left(\Delta\Pi_1^{\gamma WW}+\Delta\delta Z_{TGV}\right)\\ &-\mathcal{M}_3^+\left(\Delta\Pi_2^{\gamma WW}+\Delta\delta Z_{TGV}\right)+\frac{1}{m_W^2}\mathcal{M}_5^+\Delta\Pi_6^{\gamma WW}\Bigg]\\ &-\frac{1}{s-m_W^2}\Big[\mathcal{M}_2^+\left(\Delta\Pi_1^{ZWW}+\Delta\delta Z_{TGV}\right)\\ &-\mathcal{M}_3^+\left(\Delta\Pi_2^{ZWW}+\Delta\delta Z_{TGV}\right)+\frac{1}{m_W^2}\mathcal{M}_5^+\Delta\Pi_6^{ZWW}\Big]\Bigg\}\end{aligned} \tag{15}$$

and

$$\begin{aligned}\Delta\overline{\mathcal{M}}^{\mathrm{RL}}_{\mathrm{loop}} \doteq -\mathrm{i}e^2 2\Bigg\{\frac{1}{s}\Bigg[&\mathcal{M}_2^-\left(\Delta\Pi_1^{\gamma WW}+\Delta\delta Z_{TGV}\right)\\ &-\mathcal{M}_3^-\left(\Delta\Pi_2^{\gamma WW}+\Delta\delta Z_{TGV}\right)+\frac{1}{m_W^2}\mathcal{M}_5^-\Delta\Pi_6^{\gamma WW}\Bigg]\\ &-\frac{c_\theta}{s_\theta}g_e^-\frac{1}{s-m_W^2}\Big[\mathcal{M}_2^-\left(\Delta\Pi_1^{ZWW}+\Delta\delta Z_{TGV}\right)\\ &-\mathcal{M}_3^-\left(\Delta\Pi_2^{ZWW}+\Delta\delta Z_{TGV}\right)+\frac{1}{m_W^2}\mathcal{M}_5^-\Delta\Pi_6^{ZWW}\Big]\Bigg\},\end{aligned} \tag{16}$$

where we have employed the notation of [26], namely

$$\begin{aligned}\mathcal{M}_2^+ &\equiv \bar{v}_{\mathrm{L}}(p_1)\,\not{q}_1 u_{\mathrm{R}}(p_2)\varepsilon^*(q_1).\varepsilon^*(q_2),\\ \mathcal{M}_3^+ &\equiv \bar{v}_{\mathrm{L}}(p_1)\big[\not{\varepsilon}^*(q_1)q_1.\varepsilon^*(q_2)\\ &\qquad -\not{\varepsilon}^*(q_2)q_2.\varepsilon^*(q_1)\big]u_{\mathrm{R}}(p_2),\\ \mathcal{M}_5^+ &\equiv \bar{v}_{\mathrm{L}}(p_1)\,\not{q}_1 u_{\mathrm{R}}(p_2)\left[q_2.\varepsilon^*(q_1)\right]\left[q_1.\varepsilon^*(q_2)\right],\\ \mathcal{M}_2^- &\equiv \bar{v}_{\mathrm{R}}(p_1)\,\not{q}_1 u_{\mathrm{L}}(p_2)\varepsilon^*(q_1).\varepsilon^*(q_2),\\ \mathcal{M}_3^- &\equiv \bar{v}_{\mathrm{R}}(p_1)\big[\not{\varepsilon}^*(q_1)q_1.\varepsilon^*(q_2)\\ &\qquad -\not{\varepsilon}^*(q_2)q_2.\varepsilon^*(q_1)\big]u_{\mathrm{L}}(p_2),\\ \mathcal{M}_5^- &\equiv \bar{v}_{\mathrm{R}}(p_1)\,\not{q}_1 u_{\mathrm{L}}(p_2)\left[q_2.\varepsilon^*(q_1)\right]\left[q_1.\varepsilon^*(q_2)\right]\end{aligned} \tag{17}$$

and

$$g_e^- \equiv \frac{2s_\theta^2-1}{2s_\theta c_\theta},$$

with θ being the weak mixing angle. The LR and RL superscripts above denote the helicity configurations of the initial e^+e^- state.

The last missing piece is the tree-level amplitude $\mathcal{M}^{\mathrm{SM}}_{\mathrm{tree}}$ that can be recast in terms of these quantities as follows:

$$\mathcal{M}^{\mathrm{SM,LR}}_{\mathrm{tree}} = -\mathrm{i}e^2 2\left(\frac{1}{s}-\frac{1}{s-m_W^2}\right)\left[\mathcal{M}_2^+-\mathcal{M}_3^+\right] \tag{18}$$

and

$$\begin{aligned}\mathcal{M}^{\mathrm{SM,RL}}_{\mathrm{tree}} &= -\mathrm{i}e^2 \\ &\times\left\{2\left(\frac{1}{s}-\frac{c_\theta}{s_\theta}g_e^-\frac{1}{s-m_W^2}\right)\left[\mathcal{M}_2^--\mathcal{M}_3^-\right]\frac{1}{2s_\theta^2}\frac{1}{t}\mathcal{M}_1^-\right\}\end{aligned} \tag{19}$$

(cf. [26]). Substituting now (15)– (19) into (8) one can calculate δ for both the leptonic helicity configurations. The polarizations of the final states are encoded in the invariants (17).

5 Results and further comments

We shall present our results obtained in two "maximally" different situations: first, in the non-decoupling regime with very hierarchical heavy Higgs spectrum and then in the decoupling regime with almost degenerate heavy Higgs masses.

Our input parameters in both cases are the Higgs masses in the game (m_η, m_{h^0}, m_{H^0}, m_{A^0} and $m_{H^\pm}$). Each such set fixes four THDM parameters out of m_{12}, $\lambda_{1\ldots7}$, β and α; all the remaining ones are left to be chosen. For the sake of simplicity we shall take

$$\lambda_1=\lambda_2\equiv\lambda_{12}. \tag{20}$$

5.1 Non-decoupling regime

The result displayed in Fig. 1 is obtained within the following option:

$$\lambda_6=\lambda_7=0, \qquad m_{12}=0, \tag{21}$$

Notice that in such a case the heavy Higgs limit does not exist and therefore it is very natural to seek for non-decoupling effects in such a scenario. We have parameterized the magnitude of the heavy Higgs spectrum distortion by an overall multiplicative mass scale Λ. Due to the relations (3) and (20) one obtains

$$\lambda_{12}=\frac{m_{h^0}^2+m_{H^0}^2}{v^2}$$

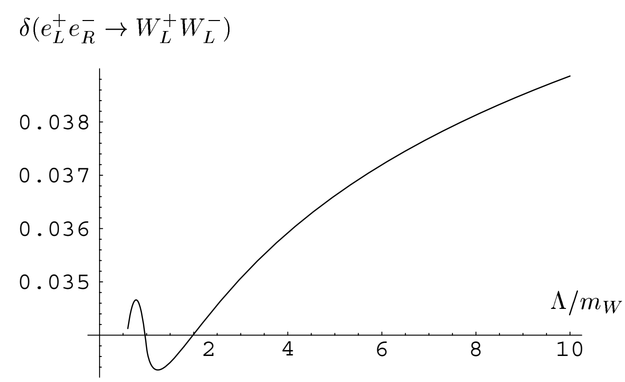


Fig. 1. δ as a function of $m_H^0=20\Lambda$, $m_{A^0}=10\Lambda$, $m_{H^\pm}=2\Lambda$. $\sqrt{s}=320\,\mathrm{GeV}$. The heavy Higgs spectrum distortion grows with Λ

In a similar manner, we can translate the λ_4, λ_5:

$$m_{H^\pm}^2 = -\frac{1}{2}\left(\lambda_4 + \lambda_5^R\right)v^2,$$
$$m_{A^\pm}^2 = -\lambda_5^R v^2.$$

The remaining parameters must obey

$$\cos^2(\alpha-\beta) = \frac{1}{m_{H^0}^2 - m_{h^0}^2} \times\Big[\left(m_{H^0}^2 + m_{h^0}^2\right)\left(1-2s_\beta^2 c_\beta^2\right) +2s_\beta^2 c_\beta^2\left(\lambda_3 v^2 - 2m_{H^\pm}^2\right) - m_{h^0}^2\Big]. \qquad (22)$$

Choosing for simplicity[1]

$$\lambda_3 = 1 \qquad \text{and} \qquad \beta \to \frac{\pi}{2}$$

the last unspecified parameter α is given by (22). Let us note that there is in fact no ambiguity in "fixing" α this way because it enters all the relevant formulae via $\cos^2(\alpha-\beta)$ only.

$\mathbf{e_L^+ e_R^- \to W_L^+ W_L^-}$:

The situation is particularly simple in the case of this polarization configuration because δ then turns out to be independent of t and u. Moreover, in the case of the longitudinally polarized vector bosons in the final state we can partially compare our results with the estimates [10] obtained by means of the equivalence theorem.

The relevant invariants $\mathcal{M}_2^-$, $\mathcal{M}_3^-$ and $\mathcal{M}_5^-$ read in this case:

$$\mathcal{M}_2^- = \frac{s-2m_W^2}{2m_W^2}\sqrt{u\,t - m_W^4},$$
$$\mathcal{M}_3^- = \frac{s}{m_W^2}\sqrt{u\,t - m_W^4},$$
$$\mathcal{M}_5^- = \frac{s(s-4m_W^2)}{4m_W^2}\sqrt{u\,t - m_W^4}.$$

Notice that the common $\sqrt{u\,t - m_W^4}$ factors cancel in the formula (8) and what remains is t and u independent. Taking into account the form of the SM tree-level amplitude (18), we see that the only relevant dynamical quantity in the game is s and therefore the deviation of THDM and SM predictions is "isotropic" (at least at the leading order).

The formulae for the formfactors $\Delta\Pi_{1,2,6}^{\gamma,ZWW}$ as well as the counterterm deviation $\Delta\delta Z_{TGV}$ are given in [18]. Due to their enormous complexity it is impossible to write down the results in a reasonably compact form. We have performed a numerical simulation in Mathematica together with the FeynCalc and LoopTools packages.

Figure 1 tells us that (for a given realization of the Higgs sector) the THDM cross section of the considered process should be enhanced by several percents with respect to its SM value and grow logarithmically with the heavy Higgs spectrum distortion Λ, in perfect agreement with what was anticipated on theoretical grounds in Sect. 2.2 and [18]. We have checked that qualitatively the same happens also in other similar setups.

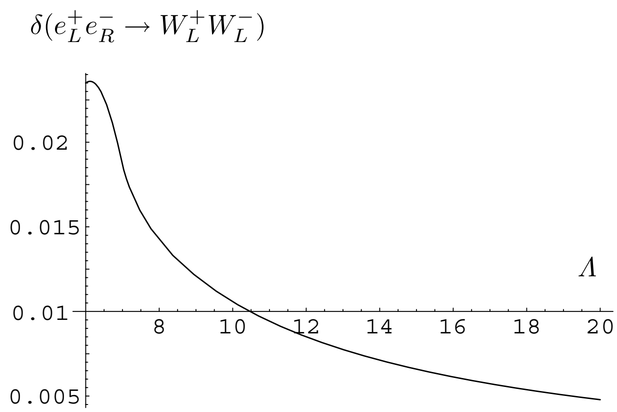


Fig. 2. δ as a function of Λ for $m_H^0 = \Lambda + 10m_W$, $m_{A^0} = \Lambda - 6m_W$, $m_{H^\pm} = \Lambda - 5m_W$. $\sqrt{s} = 200\,\mathrm{GeV}$. The heavy Higgs bosons decouple with the rising overall scale Λ since the distortion $\Delta \sim m_W$ of their spectrum remains fixed and $\Delta/\Lambda \to 0$

5.2 Decoupling regime

We achieve the *decoupling regime* by taking all the heavy Higgs masses quasidegenerate with a constant distortion $\Delta \sim m_W$ much smaller than the overall scale Λ driving the heavy part of the THDM Higgs spectrum. For $\Delta/\Lambda \to 0$ the δ should tend to 0. As can be seen in Fig. 2 this is indeed the case. This provides a non-trivial consistency check of our results. What is interesting is the fact that this picture was obtained within the setup with $m_{12} \to 0$. There is nothing that would contradict our previous considerations, because m_{12} *is not the only mass singlet parameter in the game.* There can be still big "hidden" singlets responsible for such behavior, namely the m_{11} and m_{22} just "translated" through the minimality conditions into combinations of the other parameters. This can work whenever at least one of the λ_6 or λ_7 is non-zero (justifying the choice (21) in the non-decoupling case). As we have checked, it works even better in the case of the "apparent" decoupling setup driven by the allowed non-zero m_{12}.

6 Conclusions

We have seen that within the THDM framework a non-decoupling behavior of heavy Higgs bosons can occur quite naturally. There is a simple phenomenological criterion

[1] Strictly speaking the limit $\beta \to \pi/2$ is unphysical making $\tan\beta$ infinite. What we mean is rather the setup with large $\tan\beta$ motivated by low-energy SUSY models. However, since λ_6 and λ_7 are put to zero by hand, the would-be singular behavior of $\tan\beta$ is screened and does not enter the relations of our interest.

for recognizing the character of the heavy Higgs effects in the physical amplitudes, namely the magnitude of the distortion of the heavy part of the Higgs spectrum. Since there is no possibility to get large deviations from the quasidegenerate heavy Higgs spectrum in the MSSM, the heavy Higgs bosons in the minimal supersymmetry should always decouple, which is fully compatible with the explicit analyses in the literature [25].

We have given a general one-loop estimate of such effects in the case of the physical process $e^+e^- \to W^+W^-$ within THDM. In many cases the deviation of its cross section from the SM results can be of the order of several percent, in good agreement with some previous partial analyses with longitudinally polarized vector bosons in the final state based on ET approximation. In principle, such effects could be visible in future experiments making the two Higgs doublet extension still a viable candidate of a theory beyond the standard model.

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